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Wakefield generation in one-dimensional LWFA

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Abstract. The generation of wakefields in near-critical density plasma by a short laser pulse is considered. The equations describing the plasma response to the acting laser pulse and the Hamiltonian of an electron in the wakefield are derived by using the Hamilton-Jacobi equation for the cases of $v_\mathrm{g} \simeq v_\mathrm{p} \simeq w = c$ and $w < c$ (c is the speed of light). The equations are solved analytically in two cases for a square-shaped laser pulse. The plasma potential inside the laser pulse and the wakefield potential excited by the laser pulse are obtained. The optimal length of the driving laser pulse and the wavelength of the wake are expressed by the group velocity.

PACS. 52.40.Mj Particle beam interactions in plasmas – 52.38.-r Laser-plasma interactions

1 Introduction

A laser pulse and a cold, relativistic plasma interaction process has been very attractive and interesting field of study for a long time because of its great applications in technology and science. One of the most important applications is the charged particle acceleration concept using the wakefield resulting from this interaction process. When a short laser propagates through a plasma it leaves a wake [1]. The basic plasma-based laser accelerator concepts suggest that a laser pulse propagating through a cold plasma firstly excites a large amplitude plasma wave behind itself (called wake) as a result of nonlinear interaction processes between the pulse and the plasma. The wakefield can provide a very high accelerating gradient when an electron is trapped in it. In Laser Wakefield Accelerator (LWFA) [2,3], the space-charge wave is excited by an ultrashort ($L < c/\omega_\mathrm{p}$, L is the length of the laser pulse, ω_p is the plasma frequency), intense laser pulse. When $v_\mathrm{g} = c$ (v_g is the group velocity of the laser pulse, c is the speed of light.) the largest amplitude wakefield is supposed to be produced behind the pulse.

The generation of a one-dimensional wakefield in the limiting case $v_\mathrm{g} \simeq v_\mathrm{p} \simeq c$ was studied in references [4–7] for a laser pulse with an optimal length. In ultrarelativistic case, the wakefield of the maximum amplitude is excited by a short laser pulse with a short leading front [8]. In the case, $v_\mathrm{g} \approx v_\mathrm{p} \approx w < c$, the one-dimensional wakefield generation process by multiple laser pulses was studied in terms of the wave equation for the vector potential, the Poisson equation, the continuity equation and the momentum equation in quasistatic approximation (QSA) [4] by various authors [9–11].

In this paper, we address the nonlinear generation of the one-dimensional wakefield in a cold electron plasma by a single short intense laser pulse in the Coulomb gauge. The one-dimensional approach is valid for a laser pulse of a wide front for which the characteristic time of a transverse effect, $\tau = \pi r_0^2/\lambda c$ (where r_0 is the Rayleigh length for the laser pulse), is long to compared to the characteristic time of longitudinal changes, $\tau_\mathrm{l} \approx \lambda_\mathrm{p}\gamma(n/n_0)(\lambda_\mathrm{p}/\lambda)$ (where λ and λ_p are the wavelength of the laser pulse and the plasma wave, γ is the relativistic factor). This condition is valid when $r_0 \gg \lambda_\mathrm{p}$. The pulse length is too short so that the ion motion can be neglected. Also, the electron thermal velocity is much smaller than the quiver velocity so that the plasma is cold and initially homogeneous. We assume the laser pulse is non-evolving during the interaction because the depletion of pulse is not considerable in the time $t < (8/3\sqrt{2})(\omega/\omega_\mathrm{p})(1/|a|)$ [8]. This process is studied by using the Hamilton-Jacobi equation instead of the momentum equation for cases $w \simeq c$ and $w < c$. For a strongly intense, short wave laser pulse, one can take $v_\mathrm{g} \simeq v_\mathrm{p} \simeq w < c$. The latter corresponds to the wakefield generation in near-critical density plasma by a short laser pulse. In the non-relativistic limit, a laser pulse cannot propagate in an overdense plasma, $n > n_\mathrm{c}$ (where $n_\mathrm{c} = m\omega^2/4\pi e^2$ is the critical density) due to cutoffs. For the relativistic case, recent numerical simulations show that an intense laser pulse can propagate in an overdense plasma [12,13]. From the general dispersion relation, one can easily see that this is possible for a laser pulse, $\omega^2 > \omega_\mathrm{p}^2/2\omega^2\gamma$ where $\gamma = \sqrt{1+a^2}$ and a is the vector potential of the laser light. But, this condition can be changed because for an intense laser pulse, the strong longitudinal ponderomotive force pushes electrons and this results in the increased density in front of the pulse. Moreover, γ is changed during the interaction process.

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The interaction of an intense laser pulse with plasma is accompanied by many kinds of instabilities [14] that affects the interaction process. For simplicity, instabilities are neglected in the present paper.

The paper is organized as follows. In Section 2, we study the one-dimensional description of the wakefield generation process by using the Hamilton-Jacobi equation and derive the fundamental equations, the wavebreaking field and the Hamiltonian of an electron in the wake. We obtain the analytical expression of the wakefield by a square-shaped laser pulse, the optimal length of the laser pulse giving the maximum wake, the wavelength of the excited wake wave and the plasma wave in the region, where the pulse exists, in Section 3. These are obtained for two cases; $w \simeq c$ and $w < c$. Also, the propagation condition of a laser pulse in a dense plasma is considered in this section.

2 Nonlinear description

Consider the interaction between a laser pulse and a plasma in a moving frame, $\xi = z - wt$, $\tau = t$. To derive the plasma response to this pulse, we consider the plasma being cold, uniform and with fixed immobile ion density. Throughout this paper we will use the following normalized quantities: $\phi = (e/mc^2)\varphi$, $\mathbf{a} = (e/mc^2)\mathbf{A}$, $\boldsymbol{\beta} = \mathbf{v}/c$, $\beta_w = w/c$, where φ is the plasma potential, $\mathbf{A}$ is the vector potential of the laser pulse, $\mathbf{v}$ is the velocity of an electron, e and m are the charge and the mass of the electron, respectively. Moreover, we will consider the relativistic Hamilton-Jacobi equation instead of the equation of motion for an electron.

We seek the solution of the Hamilton-Jacobi equation in the form $S = S_0(x,y,z,t)+S_1(\xi)$, where S_0 is the action of the free electron and satisfies $\mathbf{p}_0 = \boldsymbol{\nabla} S_0$, $\varepsilon_0 = -\partial S_0/\partial t$. S_1 describes the laser interaction with an electron. S is the total action for an electron in an electromagnetic field which satisfies

$$\mathbf{P} = \boldsymbol{\nabla} S,\ \varepsilon = -\frac{\partial S}{\partial t} \tag{1}$$

where $\mathbf{P}$ and ε are the generalized momentum and the energy of an electron, respectively.

After substituting this action function into the Hamilton-Jacobi equation and plugging $\varepsilon_0^2/c^2 - p_0^2 = m^2c^2$ into the resulting equation, one gets the following normalized equation for the additional action in the moving frame.

$$\left(\frac{\partial s_1}{\partial \xi}\right)^2\left(1-\frac{w^2}{c^2}\right) + 2\frac{\partial s_1}{\partial \xi}\left(\gamma_0\beta_{0z} - \beta_w - \beta_w\phi\right) + a^2 + \phi^2 + 2\left(\gamma_0\boldsymbol{\beta}_{0\perp}\cdot\mathbf{a} - \phi\right) = 0 \tag{2}$$

where $s_1 = S_1/mc$ is the normalized quantity. $\beta_z = v_z/c$ and $\boldsymbol{\beta}_\perp = \mathbf{v}_\perp/c$ are the longitudinal and the transverse normalized velocity of the electron, respectively. $\beta_{0z} = v_{0z}/c$ and $\boldsymbol{\beta}_{0\perp} = \mathbf{v}_{0\perp}/c$ are the components of the initial velocity of the electron. $\gamma = (1 - v^2/c^2)^{-1/2}$ and $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$ are the relativistic factors. We assume that the plasma quantities depend only on time and the coordinate along which it propagates and QSA is valid during the interaction process. Then the one-dimensional Poisson's equation, which describes the plasma response to the pulse, and the continuity equation in the moving frame take the following forms, respectively

$$\frac{\partial^2\phi}{\partial\xi^2} = k_\mathrm{p}^2\left(\frac{n}{n_0} - 1\right) \tag{3}$$

$$-w\frac{\partial n}{\partial \xi} + c\frac{\partial(n\beta_z)}{\partial\xi} = 0 \tag{4}$$

where $k_\mathrm{p} = \omega_\mathrm{p}/c$, $\omega_\mathrm{p} = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency, n_0 is the background ion density, n is the plasma electron density.

The Coulomb gauge gives $\partial a_z/\partial z = 0$ since the vector potential is independent of x and y. This implies $a_z = 0$ due to the wave equation in vacuum. Therefore, from equation (1) one can get

$$\gamma\boldsymbol{\beta}_\perp = \gamma_0\boldsymbol{\beta}_{0\perp} + \mathbf{a}, \tag{5}$$

$$\gamma\beta_z = \gamma_0\beta_{0z} + \beta_w\frac{\partial s_1}{\partial\xi}, \tag{6}$$

$$\frac{\varepsilon}{mc^2} = -\gamma_0 + \beta_w\frac{\partial s_1}{\partial \xi}\cdot \tag{7}$$

The equation (5) implies that the transverse quiver momentum is conserved in the one-dimensional limit. It enables to express the relativistic factor as

$$\gamma = \sqrt{\frac{1+(\boldsymbol{\beta}_{0\perp}+\mathbf{a})^2}{1-\beta_z^2}}\cdot \tag{8}$$

Using equations (2, 6, 8) one can find the normalized velocity as

$$\beta_z = \frac{\beta_w - \sqrt{1-(1-\beta_w^2)\frac{1+a^2}{(1+\phi)^2}}}{1-\beta_w\sqrt{1-(1-\beta_w^2)\frac{1+a^2}{(1+\phi)^2}}}\cdot \tag{9}$$

Here, we assumed $\gamma_0 = 1$ and $\beta_0 = 0$.

From equation (7) and the energy expression, $\varepsilon = \gamma mc^2 - e\varphi$, for an electron in an electromagnetic field, one can obtain that the quantity

$$\gamma(1-\beta_w\beta_z) - \phi = \mathrm{const} \tag{10}$$

is conserved during the interaction process. This is the Hamiltonian of a single electron in the one-dimensional wakefield and can be used to study the acceleration process when $a^2 = 0$.

From equations (9, 4) the Poisson's equation is written in the following form

$$\frac{\partial^2\phi}{\partial\xi^2} = \frac{k_\mathrm{p}^2}{1-\beta_w^2}\left(\frac{\beta_w}{\sqrt{1-(1-\beta_w^2)\frac{1+a^2}{(1+\phi)^2}}} - 1\right)\cdot \tag{11}$$

Equation (11) is the same as derived by Sprangle and his group [9], in which it is used to analyze the harmonic generations and used in the study of Resonantly-driven Laser Wake Accelerator [10,11] and in obtaining the enhanced wakefield by Kingham and Bell [15].

In order that this equation has a real solution, the first term in the parenthesis on the right hand side of equation (11) must be real *i.e.* $(1+\phi)^2-\left(1-\beta_w^2\right)\left(1+a^2\right)\geq 0$ and the minimum value of the potential to give a real solution is $\phi_{\min}=\sqrt{(1-\beta_w^2)\left(1+a^2\right)}-1$. This corresponds to the wavebreaking electric field

$$E=\sqrt{2}\left(\frac{1}{\sqrt{1-\beta_w^2}}-1\right)^{1/2}.$$

We note that the energy of randomly distributed electrons in the wake wave can exceed the energy corresponding to wavebreaking field. These electrons may form an electron beam of high energies.

Consider the limiting case, $\beta_w \approx 1$. Since $\sqrt{1-\beta_w^2}\ll 1$, one can expand the right hand side of equation (11). This gives the following equation

$$\frac{\partial^2\phi}{\partial\xi^2}=\frac{1}{2}k_{\mathrm{p}}^2\left(\frac{1+a^2}{(1+\phi)^2}-1\right). \tag{12}$$

This equation describes the plasma response to the laser pulse for $w\simeq c$. For a circularly polarized square-shaped laser pulse, $a^2=a_0^2$.

In the derivation of above equations (obviously, manipulation of the Hamilton-Jacobi equation), the envelope of the laser pulse was assumed to be non-evolving and is a function of ξ only.

3 Analytical solution

3.1 The case $\mathrm{v_g}\simeq\mathrm{v_p}\simeq\mathrm{c}$

Equation (12) with the natural initial conditions, $\phi(0)=0$, $\phi'(0)=0$, describes the plasma potential in the region where laser pulse exists in limiting case $w\simeq c$. The values of the plasma potential, within a range 1 and $\gamma_\perp^2\equiv 1+a_0^2$, and the longitudinal electric field at the end of the driving laser pulse determines the initial values of the wake. So, the maximum and the minimum value of the wake amplitude depend on these values. Changing the power, the length and the group velocity of the driving pulse, one can drive these values at the pulse end.

We consider the general case for a square-shaped laser pulse. Equation (11) with the appropriate initial condition describes the wake potential when $a^2=0$. For simplicity, we introduce the new variables, $\Phi=\phi+1$, $\zeta=k_{\mathrm{p}}\xi$. When the plasma potential takes an intermediate value, Φ_0 between 1 and $\gamma_\perp^2$ at the end of the pulse, the boundary conditions $\Phi(L)=\Phi_0$, $\acute{\Phi}'(L)=\sqrt{(\Phi_0-1)(\gamma_\perp^2-\Phi_0)/\Phi_0}$ are valid for the wake potential and there exists β_1, β_2 satisfying

$$\begin{cases}\beta_1+\beta_2=\dfrac{(\Phi_0-1)(\gamma_\perp^2-\Phi_0)}{\Phi_0}+\dfrac{\Phi_0^2+1}{\Phi_0}\\ \beta_1\beta_2=1\end{cases}.$$

In this case the wake potential is determined as

$$\zeta=L-2\sqrt{\beta_2}E(m,\chi) \tag{13}$$

where $\chi=\arcsin\sqrt{(\beta_2-\phi)/(\beta_2-\beta_1)}$, $m=\sqrt{1-\beta_1/\beta_2}$. The wavelength of this wake wave and the length of the laser pulse are $\lambda=4\sqrt{\beta_2}E(m)$ and $L=2\gamma_\perp E(p)-2\gamma_\perp E(p,\alpha)$ (where $p^2=1-1/\gamma_\perp^2$, $\sin^2\alpha=(\gamma_\perp^2-\phi_0)/(\gamma_\perp^2-1)$, respectively.

This solution is valid when $\beta_1\leq\Phi<\beta_2$ [15], whereas the results in references [5,7] were taken in $\beta_1<\Phi\leq\beta_2$ for the optimal laser pulse that is the pulse giving the maximum wake at its end. When $\Phi_0=\gamma_\perp^2$ or $\beta_1=1/\gamma_\perp^2$ and $\beta_2=\gamma_\perp^2$, this solution gives the wakefield generated by the optimal laser pulse and the corresponding laser pulse length and plasma wave length match to well known results. The case $\beta_1=\beta_2=1$ corresponds to the wakeless regime [7]. When the plasma potential takes an intermediate value Φ_0 between 1 and $\gamma_\perp^2$ at the end of the pulse, the condition of the wakeless regime $L=\lambda_{\mathrm{p}}$ [7] is not satisfied. In contrary, this condition is for the maximum wake wave.

3.2 The case $\mathrm{v_g}\approx\mathrm{v_p}\approx\mathrm{w}<\mathrm{c}$

This case corresponds to the wakefield in the near-critical density plasma. Although, an electromagnetic wave cannot propagate in an overdense plasma, for an ultrarelativistic intense laser pulse it is possible because of the relativistic ponderomotive force [13]. The general expression of the refractive index of a cold plasma for a short intense pulse is

$$\eta\approx 1-\frac{\omega_{\mathrm{p}}^2}{\omega^2}\beta_{\mathrm{p}}\frac{k_{\mathrm{p}}^2}{1+\phi}\left(1-(1-\beta_{\mathrm{p}}^2)\frac{1+a^2}{(1+\phi)^2}\right)^{-2}\quad[14].$$

Therefore, the propagation occurs when

$$\frac{\omega^2}{\omega_{\mathrm{p}}^2}>\beta_{\mathrm{p}}\frac{k_{\mathrm{p}}^2}{1+\phi}\left(1-(1-\beta_{\mathrm{p}}^2)\frac{1+a^2}{(1+\phi)^2}\right)^{-2}.$$

This condition shows the possibility of the propagation of a wave in near-critical density plasma. In this case, equation (11) describes the plasma response and cannot be integrated analytically for an arbitrary shape of pulses. We consider a square-shaped pulse. For a circularly polarized square-shaped laser pulse $a^2=a_0^2$.

On this equation, the Cauchy's initial condition, $\Phi(0)=1$, $\Phi'(0)=0$ are imposed. The first integration of equation (11) with this conditions gives the following first order differential equation

$$\frac{1}{2}(1-\beta_w^2)\left(\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta}\right)^2=1-\Phi+\beta_w\left(\sqrt{\Phi^2-(1-\beta_w^2)\left(1+a_0^2\right)}-\sqrt{1-(1-\beta_w^2)\left(1+a_0^2\right)}\right) \tag{14}$$

By using the substitution

$$x^2 = 2\left(\sqrt{\Phi^2 - (1-\beta_w^2)(1+a_0^2)} + \Phi\right),$$

this equation is integrated to give

$$\zeta = \sqrt{\frac{1+\beta_w}{2}}\left[x_1 E(\chi_1, q_1) - \frac{1}{x}\sqrt{(x_1^2 - x^2)(x^2 - x_2^2)} - \frac{4(1-\beta_w^2)(1+a_0^2)}{x_1 x_2^2} E(\chi_1, q_1)\right]$$

where $E(\chi_1, q_1)$ is the incomplete elliptic integral of the second kind,

$$\chi_1 = \arcsin \frac{x_1}{x}\sqrt{\frac{x^2 - x_2^2}{x_1^2 - x_2^2}},$$

$$q_1 = \sqrt{\frac{x_1^2 - x_2^2}{x_1^2}},$$

$$x_1^2 = 2\left(\frac{1+\beta_w}{1-\beta_w}\right)\left(1 - \sqrt{1-(1-\beta_w^2)(1+a_0^2)}\right),$$

and

$$x_2^2 = 2\left(1 + \sqrt{1-(1-\beta_w^2)(1+a_0^2)}\right).$$

Using the boundary conditions above in the result of integration, the optimal length of a laser pulse creating the maximum potential at $\zeta = -L_{\text{op}}$ can be obtained

$$L_{\text{op}} = \sqrt{\frac{1+\beta_w}{2}}\left[\frac{4(1-\beta_w^2)(1+a_0^2)}{x_1 x_2^2} - x_1\right] E\left(\frac{\pi}{2}, q_1\right) \quad (15)$$

where $E(\pi/2, q_1)$ is the complete elliptic integral of the second kind.

Now, consider the wake potential. At a given value of the amplitude of the square-shaped laser pulse, the maximum longitudinal electric field corresponds to $\Phi = 1$. Assuming that the potential takes the maximum value at $\zeta = -L_{\text{op}}$, one can impose the following boundary condition on this potential for the optimal laser pulse, $\Phi(-L_{\text{op}}) = \Phi_1$, $\Phi'(-L_{\text{op}}) = 0$ where

$$\Phi_1 = \frac{1+\beta_w^2 - 2\beta_w\sqrt{1-(1-\beta_w^2)(1+a_0^2)}}{1-\beta_w^2}.$$

Similarly, as we have done above, with the substitution

$$y^2 = 2\left(\sqrt{\Phi^2 - (1-\beta_w^2)} + \Phi\right),$$

this equation is integrated to give

$$\zeta = \sqrt{\frac{1+\beta_w}{2}}\left[-L_{\text{op}} + y_2 E(\chi_2, q_2) - \frac{1}{y}\sqrt{(y_2^2 - y^2)(y^2 - y_1^2)} - \frac{4(1-\beta_w^2)}{y_2 y_1^2} E(\chi_2, q_2)\right] \quad (16)$$

where

$$\chi_2 = \arcsin\sqrt{\frac{y^2 - y_1^2}{y_2^2 - y_1^2}},$$

$$q_2 = \sqrt{\frac{y_2^2 - y_1^2}{y_2^2}},$$

$$y_1^2 = 2\left(\Phi_1 + \sqrt{\Phi_1^2 - (1-\beta_w^2)}\right),$$

$$y_2^2 = 2\left(\frac{1+\beta_w}{1-\beta_w}\right)\left(\Phi_1 - \sqrt{\Phi_1^2 - (1-\beta_w^2)}\right).$$

From the result of integration one can find the wavelength of the wake wave as

$$\lambda = \sqrt{2(1+\beta_w)}\left[y_2 - \frac{4(1-\beta_w^2)}{y_2 y_1^2}\right] E\left(\frac{\pi}{2}, q_2\right). \quad (17)$$

The results for a limiting case can be taken from these results setting $w \approx c$ or $\sqrt{1-w^2/c^2} \ll 1$ in the corresponding expressions. In this case, $x_1 \approx 2\gamma_\perp^2$, $x_2 = 2$, $q_1 \approx m$, and so, the optimal length of the driving laser pulse and the wave length of the plasma wake wave become $L_{\text{op}} \approx 2\gamma_\perp E(\pi/2, m)$, $\lambda \approx 2\gamma E(\pi/2, q_2)$. The wavelength becomes shorter than that of the case, $w \approx c$. When $w \ll c$, the solution does not exist and implies that the propagation is impossible.

4 Conclusion

We have obtained equations (11, 12) using the Hamilton-Jacobi equation, not integrating the momentum equation. It is shown that employing the Hamilton-Jacobi equation in the one-dimensional description of the laser plasma interaction is more direct method to reach the purpose. In this description, the Hamiltonian of an electron in a plasma wave and the wavebreaking field are easily obtained. The equation (11) is used to analyze the nonlinear wake wave generation by a single short laser pulse. This consideration is valid for a plasma with

$$\omega_{\text{p}}^2 > \omega^2 > \omega_{\text{p}}^2 \beta_{\text{p}} \frac{k_{\text{p}}^2}{1+\phi}\left(1 - (1-\beta_{\text{p}}^2)\frac{1+a^2}{(1+\phi)^2}\right)^{-2}.$$

The excited plasma waves strongly depend on the intensity, the group velocity and the length of the laser pulse. When $w \approx c$, the general expression of the wake by a square-shaped laser pulse are taken and this enables to study the wake when the plasma wave amplitude within the pulse takes the intermediate value $(1, \gamma_\perp^2)$ at the end of laser pulse.

The Hamilton-Jacobi equation can be successfully used in the study of the trapping of electrons in the wake wave.

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